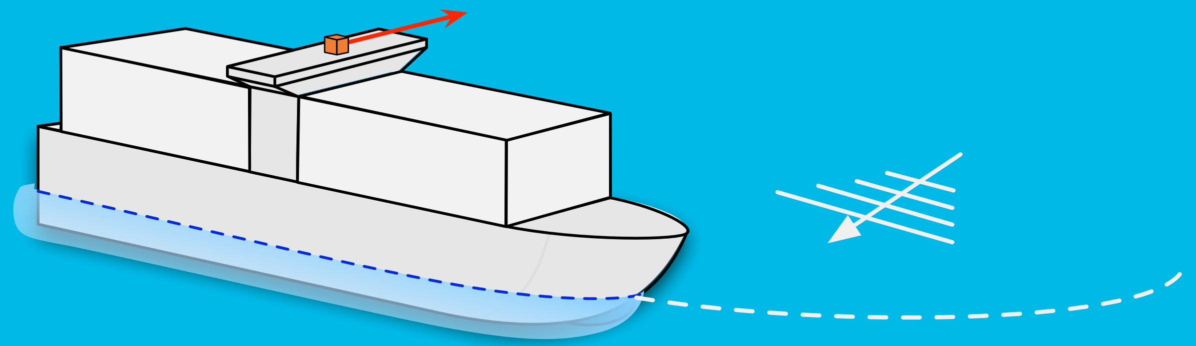
Identification of Systems with Unknown Inputs
Using Indirect Input Measurements

Jonas Linder and Martin Enqvist



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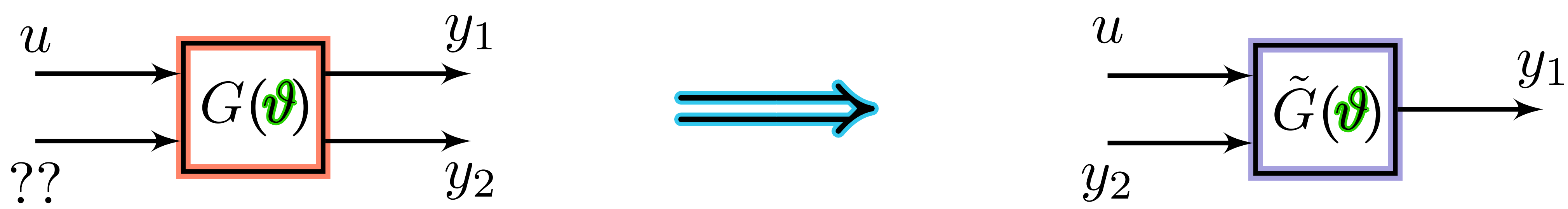
Overview

A framework for systematically reformulating a model with unknown inputs into a model with known inputs that can be used to estimate the desired dynamics



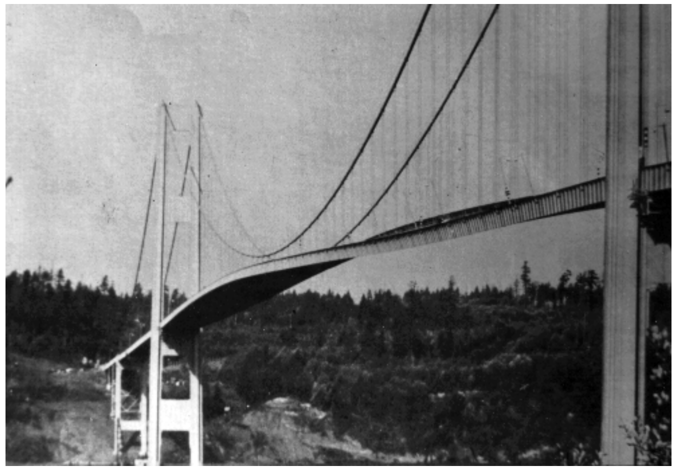
$\hat{\vartheta} = ?$ $\hat{\vartheta} = \vartheta_0$ [1]

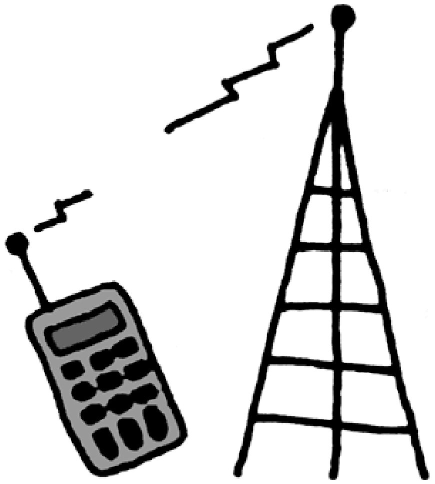
[1] Sometimes :)

Motivation - Systems with unknown inputs

Examples

- Robotics – varying payload
- Automotive industry – slope of road
- Marine applications – wind and wave
- Bridges and planes – vibration in structure
- Telecommunication – unknown data and channel



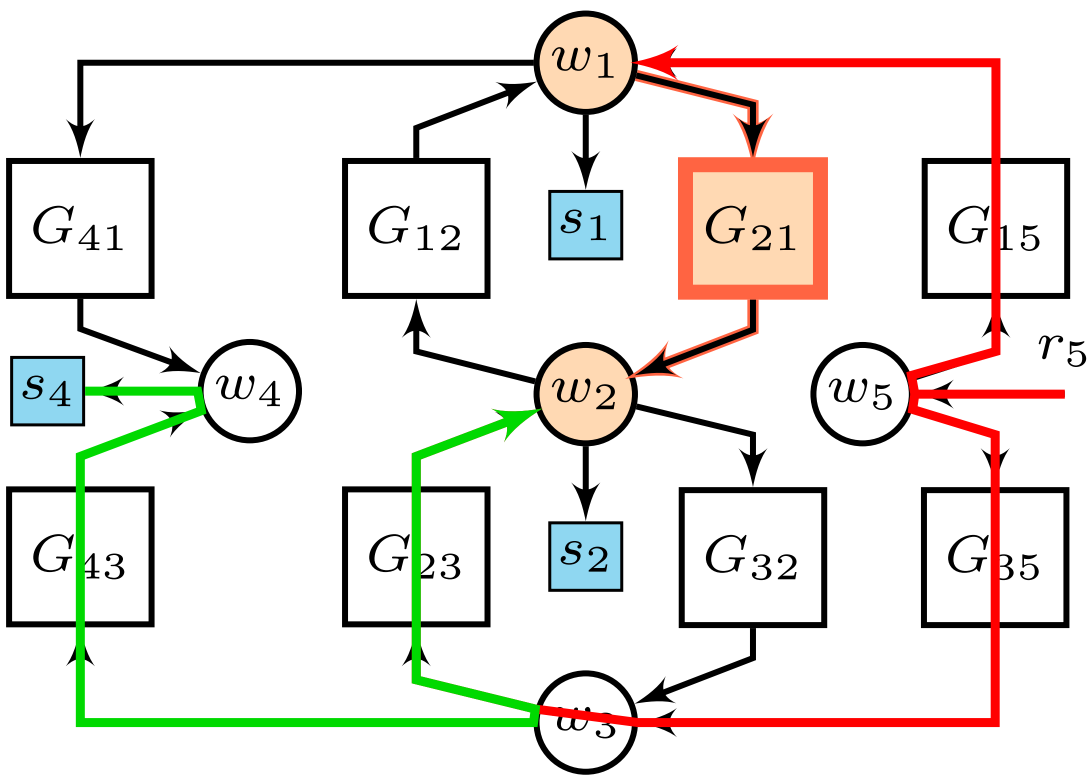


Remedies for unknown inputs

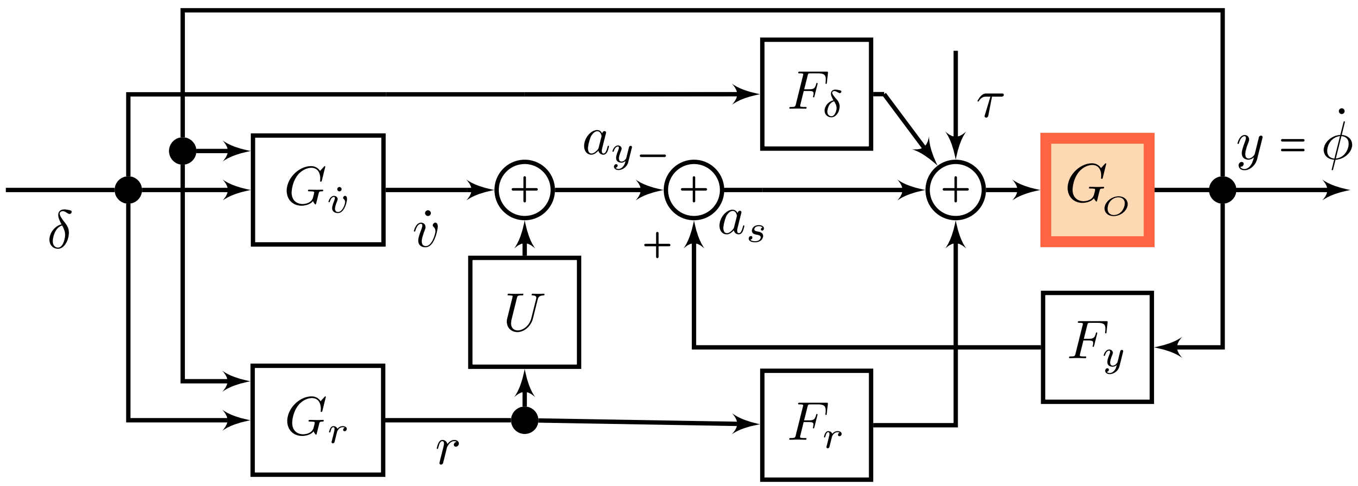
Indirect framework {
1. Neglect or partially mitigate
2. Measure – EIV
3. Eliminate – sensor-to-sensor

4. Assume – time-series, blind identification

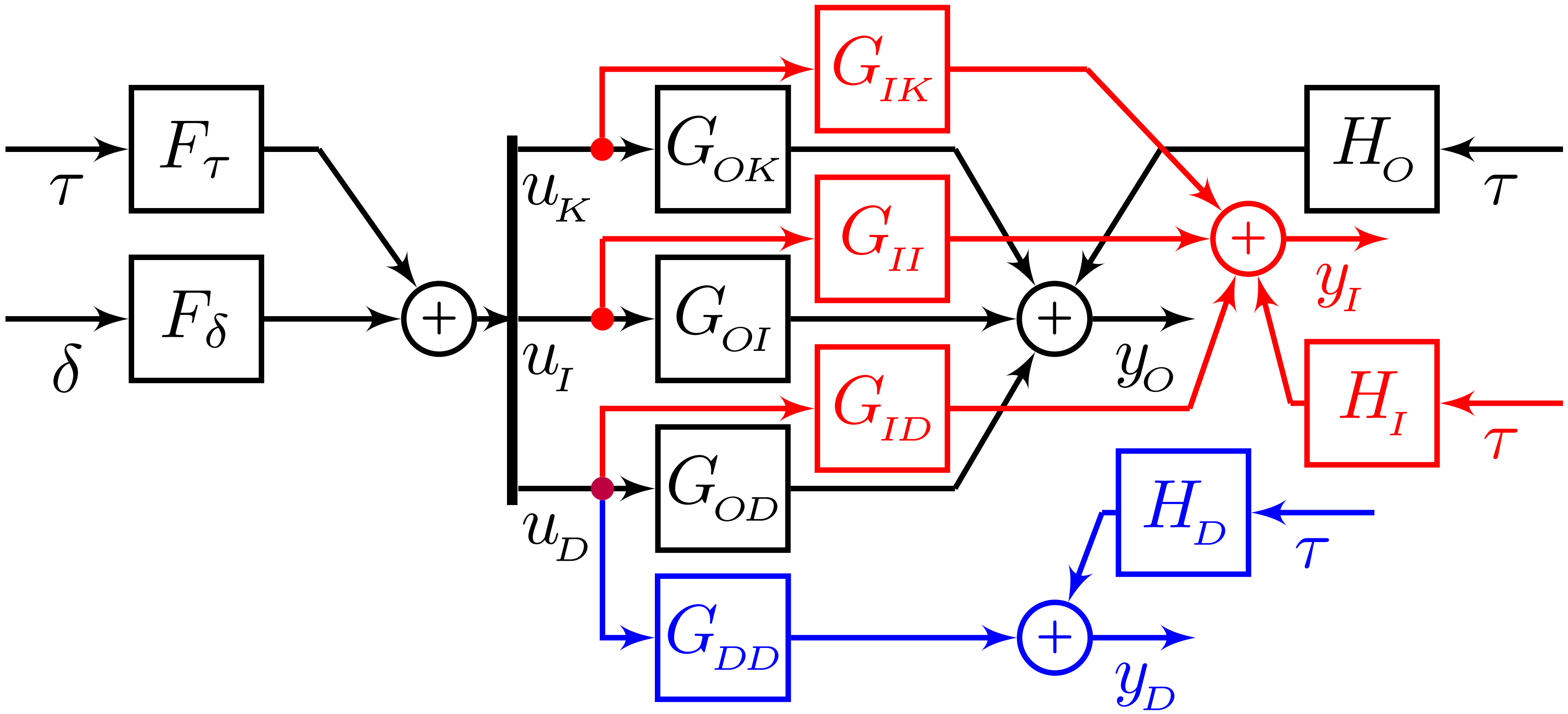
Motivation - Illustrative example



- Complete model intractable
- Estimate part of system
- Limited measurements s_x
- Correlation $\Longrightarrow$ bias
- Indirect input measurements
- Unconventional networks



Problem formulation



- Estimate the MIMO transfer function $G_O(\mathrm{p})$ from

$$y_O = G_O(\mathrm{p})u + H_O(\mathrm{p})\tau \quad \text{and} \quad u = F_\delta \delta + F_\tau \tau$$

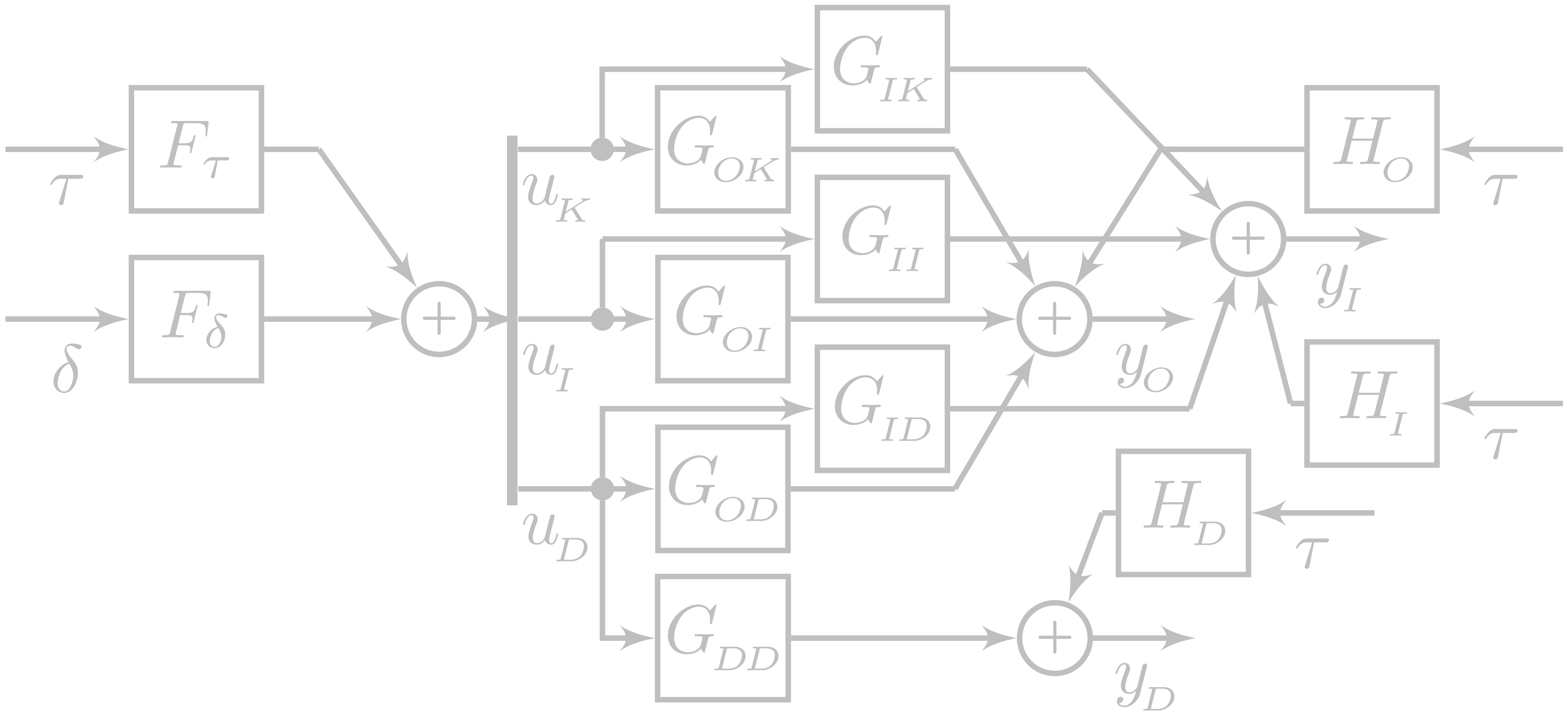
- The input is assumed to be (partially) unknown

$$u = \begin{bmatrix} u_K \\ u_I \\ u_D \end{bmatrix} \begin{matrix} \} \text{(exactly) known} \\ \Big\} \text{unknown} \end{matrix}$$

- Extend model with the direct- and indirect input measurements

$$\begin{bmatrix} y_O \\ y_I \\ y_D \end{bmatrix} = \begin{bmatrix} G_{OK} & G_{OI} & G_{OD} \\ G_{IK} & G_{II} & G_{ID} \\ 0 & 0 & G_{DD} \end{bmatrix} \begin{bmatrix} u_K \\ u_I \\ u_D \end{bmatrix} + \begin{bmatrix} H_O \\ H_I \\ H_D \end{bmatrix} \tau$$

Problem formulation - Indirect model



- **Idea:** Eliminate unknown input using **algebraic manipulation**
- **Assumption:** There exist filters f_{DD} and f_{II}, such that $f_{DD}G_{DD} = I$ and $f_{II}G_{II} = I$
- Solve for $G_{DD}u_D$ and $G_{II}u_I$ and apply filters

$$u_D = f_{DD}\left[y_D - H_D\tau\right] \qquad u_I = f_{II}\left[y_I - G_{IK}u_K - G_{ID}u_D - H_I\tau\right]$$

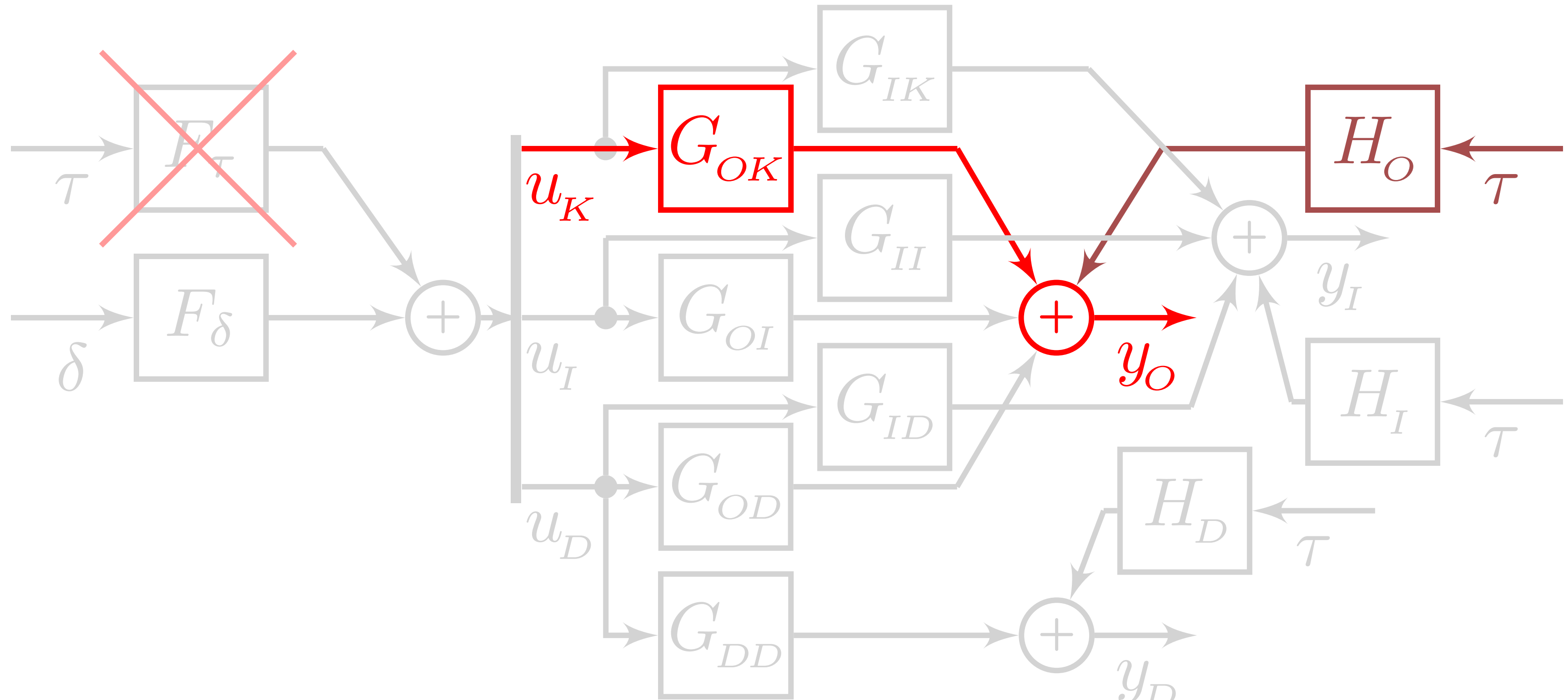
- The indirect model

$$y_O = \left[\tilde{G}_{OK}u_K + \tilde{G}_{OI}y_I + \tilde{G}_{OD}y_D\right] + \bar{\tau} = \tilde{G}_O\tilde{u} + \bar{\tau}$$

- Extend model with the direct- and indirect input measurements

$$\begin{bmatrix} y_O \\ y_I \\ y_D \end{bmatrix} = \begin{bmatrix} G_{OK} & G_{OI} & G_{OD} \\ G_{IK} & G_{II} & G_{ID} \\ 0 & 0 & G_{DD} \end{bmatrix}\begin{bmatrix} u_K \\ u_I \\ u_D \end{bmatrix} + \begin{bmatrix} H_O \\ H_I \\ H_D \end{bmatrix}\tau$$

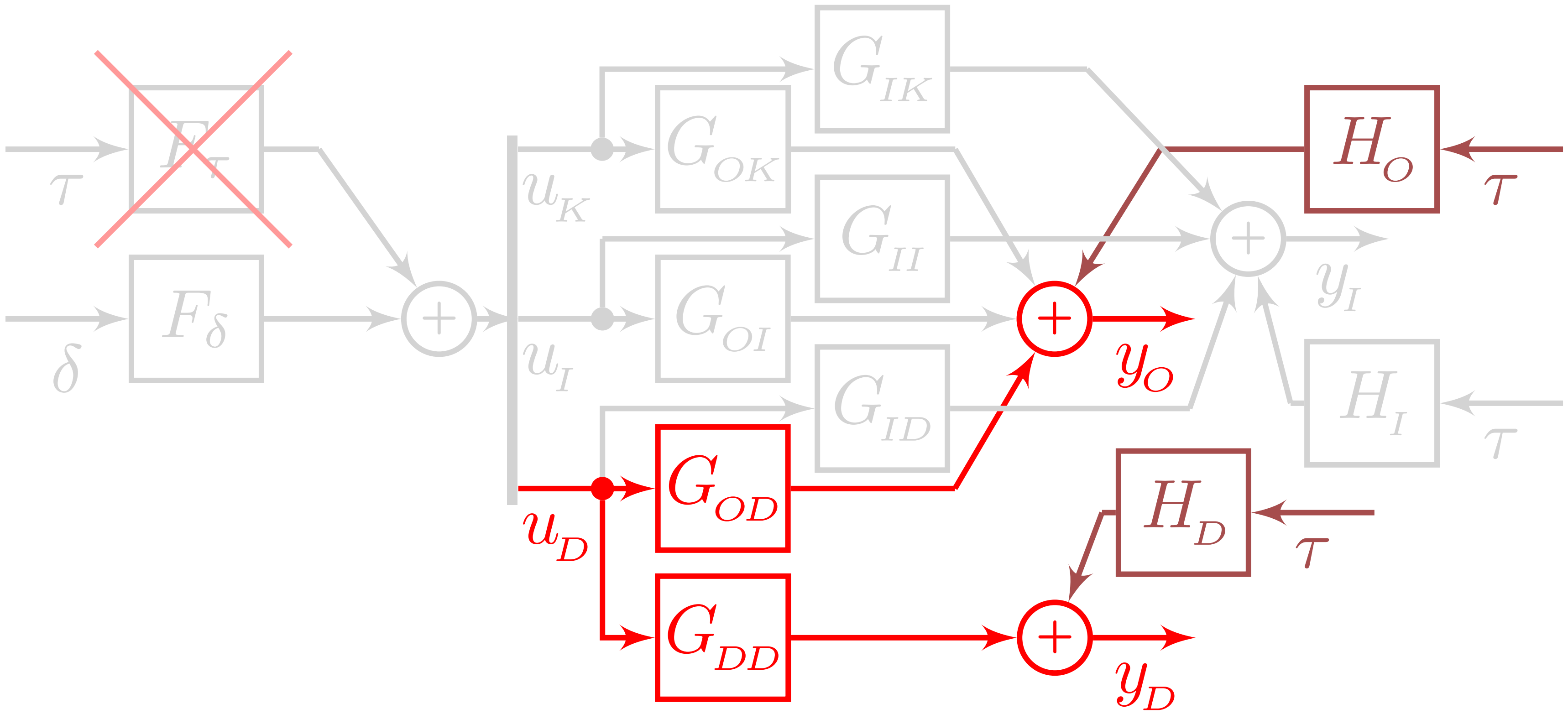
Connection to previous work – Open-loop configuration



$$y_O = \left[G_{OK} - G_{OI}G_{II}^{-1}G_{IK}\right]u_K + \left[G_{OI}G_{II}^{-1}\right]y_I + \left[G_{OD} - G_{OI}G_{II}^{-1}G_{ID}\right]G_{DD}^{-1}y_D + \bar{\bar{\tau}}$$

- Input *uncorrelated* with the process disturbance, i.e. $F_\tau = 0$
  - All inputs known: Classic open-loop

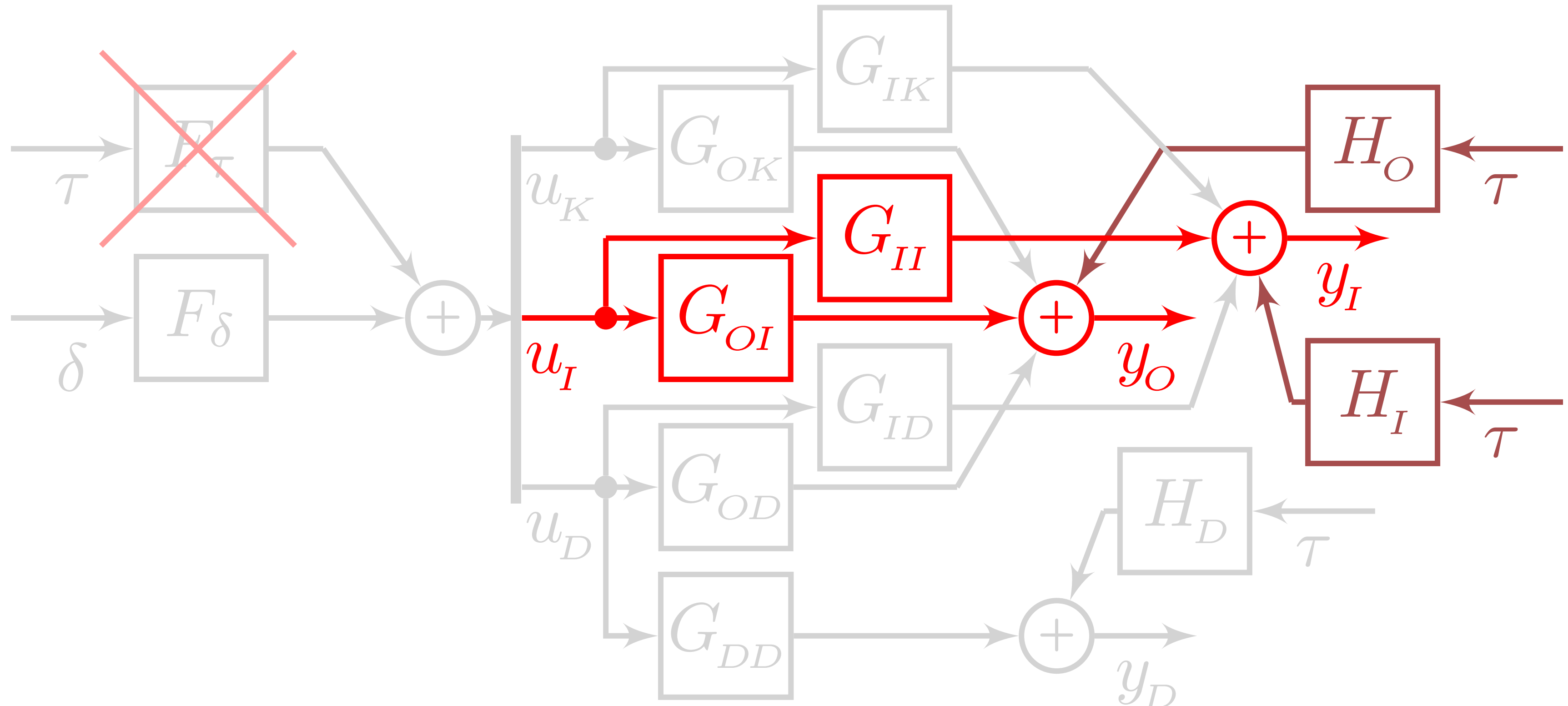
Connection to previous work – Open-loop configuration



$$y_O = \left[G_{OK} - G_{OI}G_{II}^{-1}G_{IK}\right]u_K + \left[G_{OI}G_{II}^{-1}\right]y_I + \left[G_{OD} - G_{OI}G_{II}^{-1}G_{ID}\right]G_{DD}^{-1}y_D + \bar{\tau}$$

- Input *uncorrelated* with the process disturbance, i.e. $F_\tau = 0$
  - All inputs known: Classic open-loop
  - All inputs directly measured: Classic errors-in-variable ($G_{DD} = I$ and $e_D = H_D\tau$)

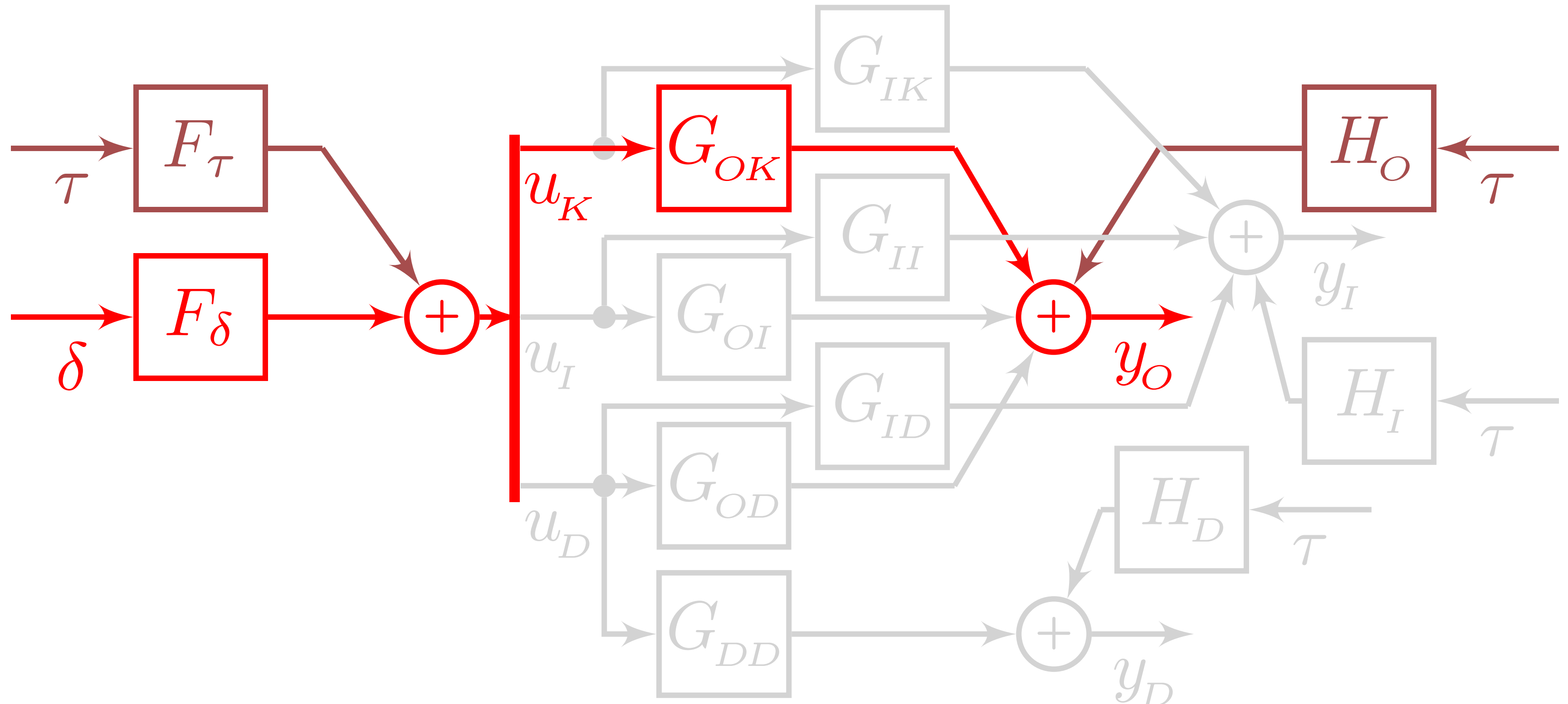
Connection to previous work – Open-loop configuration



$$y_O = \left[G_{OK} - G_{OI}G_{II}^{-1}G_{IK}\right]u_K + \left[G_{OI}G_{II}^{-1}\right]y_I + \left[G_{OD} - G_{OI}G_{II}^{-1}G_{ID}\right]G_{DD}^{-1}y_D + \bar{\tau}$$

- Input *uncorrelated* with the process disturbance, i.e. $F_\tau = 0$
  - All inputs known: Classic open-loop
  - All inputs directly measured: Classic errors-in-variable ($G_{DD} = I$ and $e_D = H_D\tau$)
  - All inputs indirectly measured: Sensor-to-sensor system identification problem

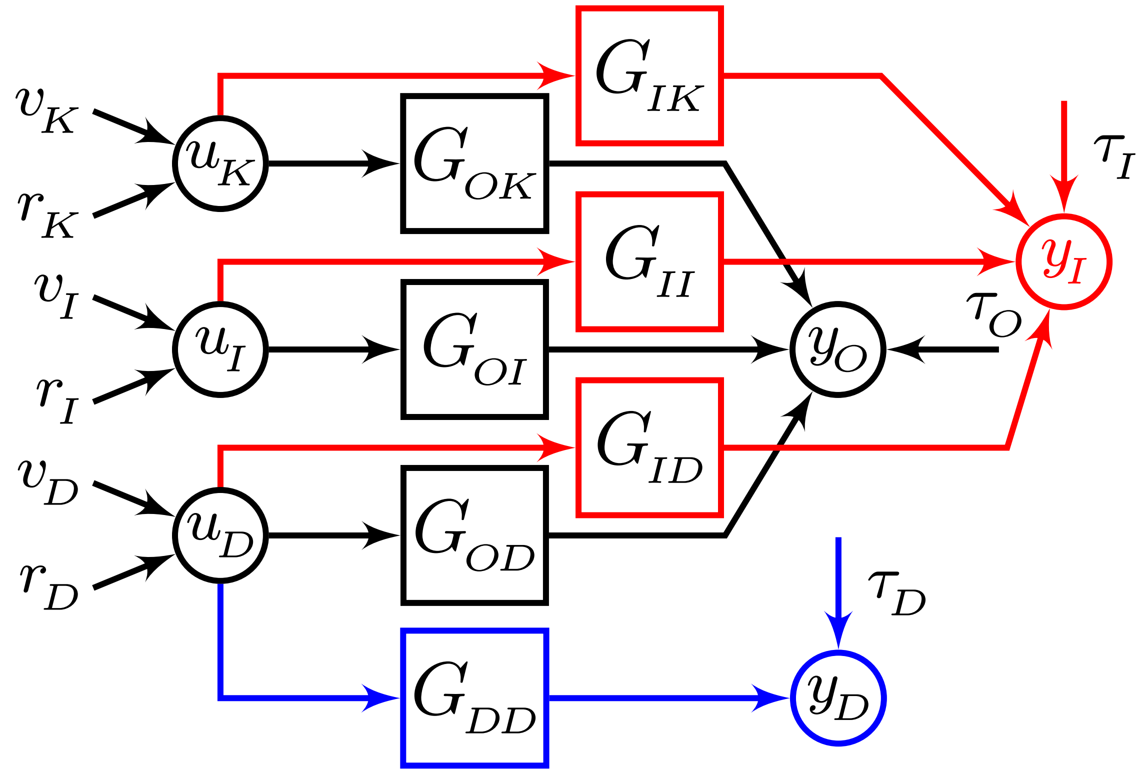
Connection to previous work – Closed-loop configuration



$$y_O = \left[G_{OK} - G_{OI}G_{II}^{-1}G_{IK}\right]u_K + \left[G_{OI}G_{II}^{-1}\right]y_I + \left[G_{OD} - G_{OI}G_{II}^{-1}G_{ID}\right]G_{DD}^{-1}y_D + \bar{\tau}$$

- Input *uncorrelated* with the process disturbance, i.e. $F_\tau = 0$
  - All inputs known: Classic open-loop
  - All inputs directly measured: Classic errors-in-variable ($G_{DD} = I$ and $e_D = H_D\tau$)
  - All inputs indirectly measured: Sensor-to-sensor system identification problem
- Input *correlated* with the process disturbance, i.e. $F_\tau \neq 0$
  - All inputs known: Classic closed-loop

Connection to previous work - Closed-loop configuration



$$y_O = \left[G_{OK} - G_{OI}G_{II}^{-1}G_{IK}\right]u_K + \left[G_{OI}G_{II}^{-1}\right]y_I + \left[G_{OD} - G_{OI}G_{II}^{-1}G_{ID}\right]G_{DD}^{-1}y_D + \bar{\tau}$$

- Input *correlated* with the process disturbance, i.e. $F_\tau \neq 0$ (continued)
  - Dynamic networks where the notation $v = F_\tau \tau$ and $r = F_\delta \delta$ is used

Generality of the Indirect Framework

- Both F_τ and F_δ can be full matrices (allows correlation between external signals)
- The indirect model “flips the arrows” to utilize the input measurements

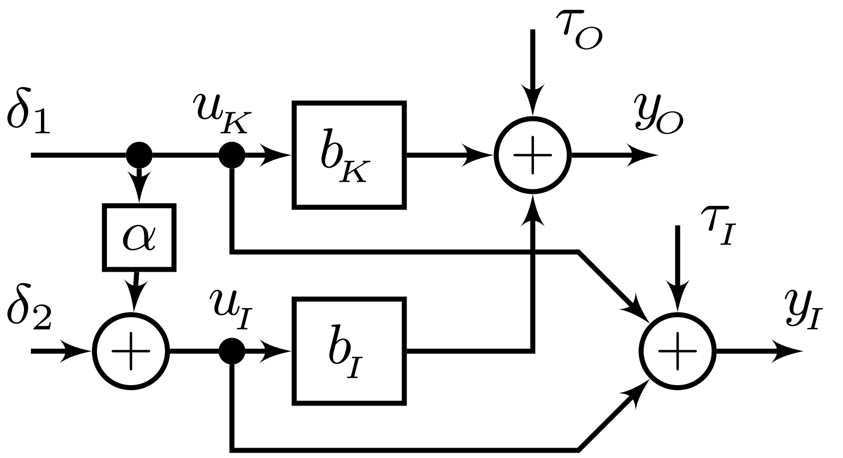
Benefits of the framework

Why is it necessary or beneficial to include an input?

- Identifiability – can both be gained and lost
- Consistency and Variance

Benefits of the framework - Consistency & variance

- An input can typically be neglected without affecting the consistency if it is independent of the remaining inputs.
- It is beneficial to use the input measurement if the signal-to-noise ratio is high.



Example

True system

$$y_O = b_K u_K + b_I u_I + \tau_O$$

$$y_I = u_K + u_I + \tau_I$$

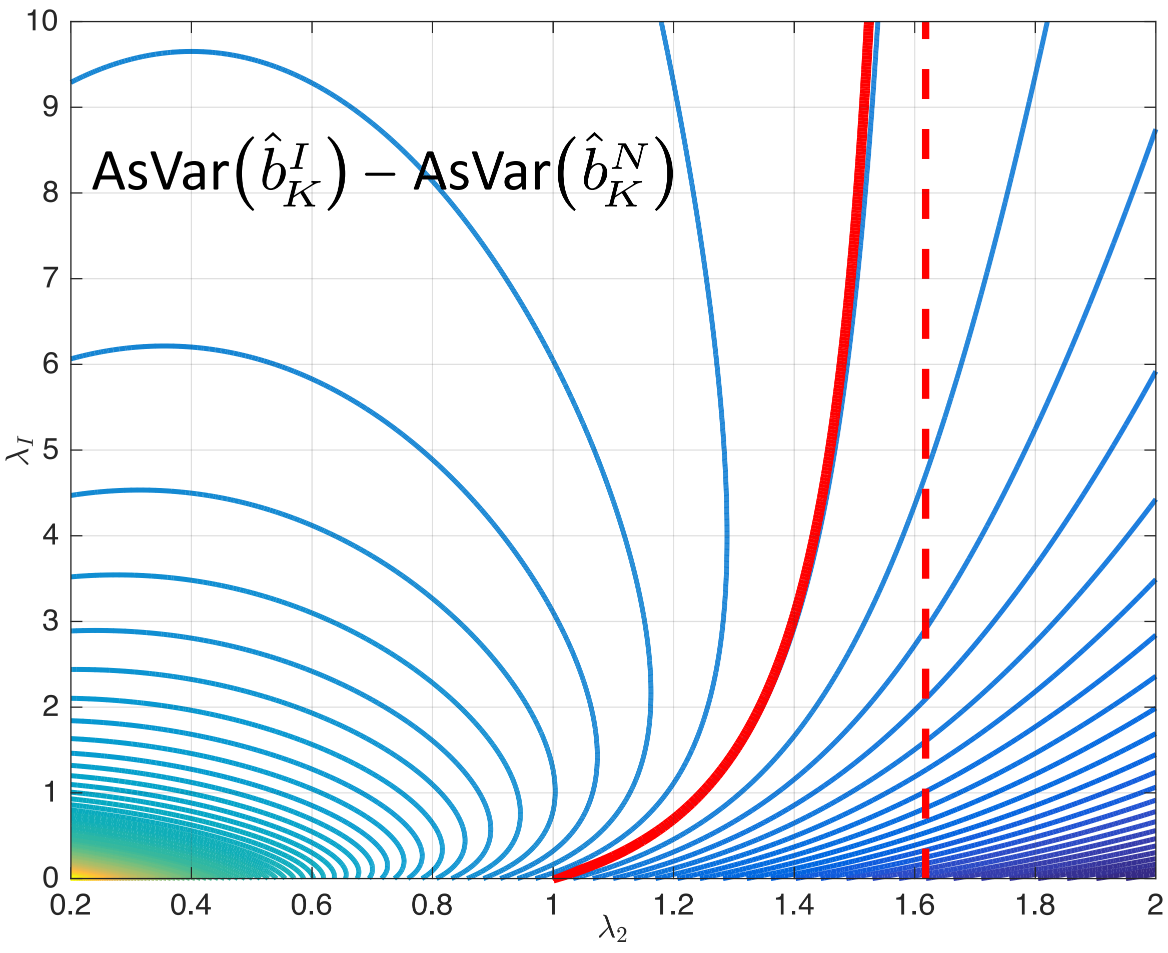
where we are interested in b_K .

Predictor with neglected input

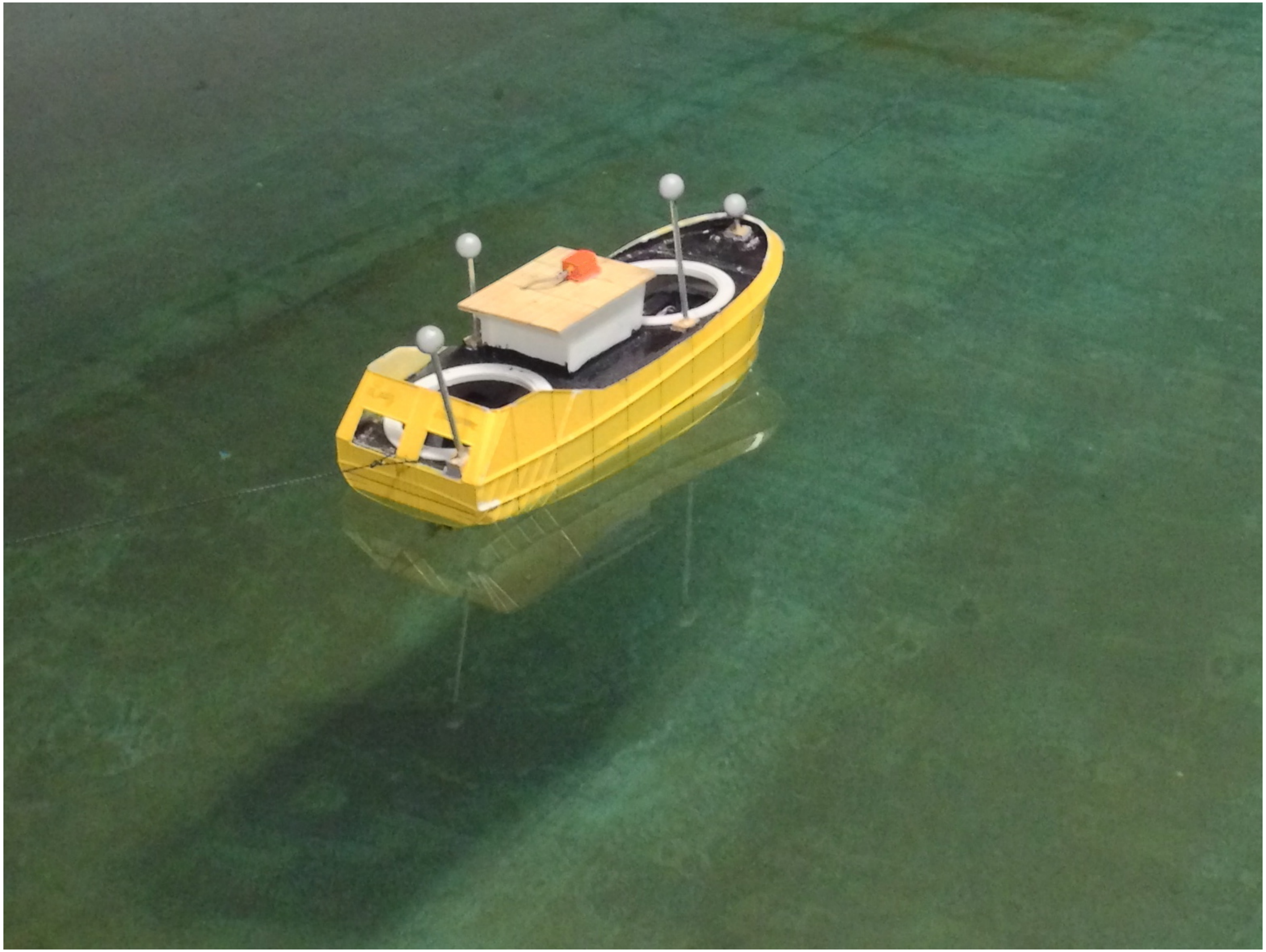
$$\hat{y}_O = \hat{b}_K^N u_K \Rightarrow b_K^* = b_K + \alpha b_I$$

Predictor with using input measurement

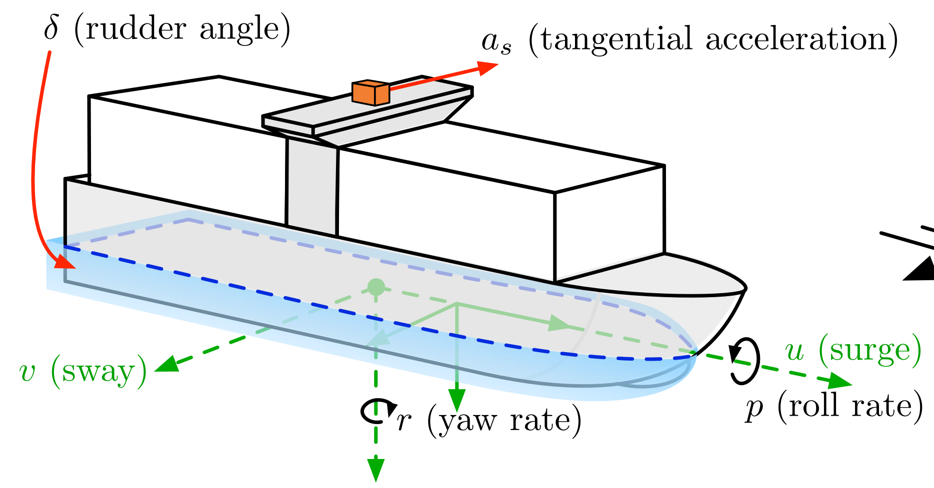
$$\hat{y}_O = (\hat{b}_K^I - \hat{b}_I) u_K + \hat{b}_I y_I \Rightarrow b_K^* = b_K + \alpha b_I \frac{\lambda_I}{\lambda_2 + \lambda_I}$$

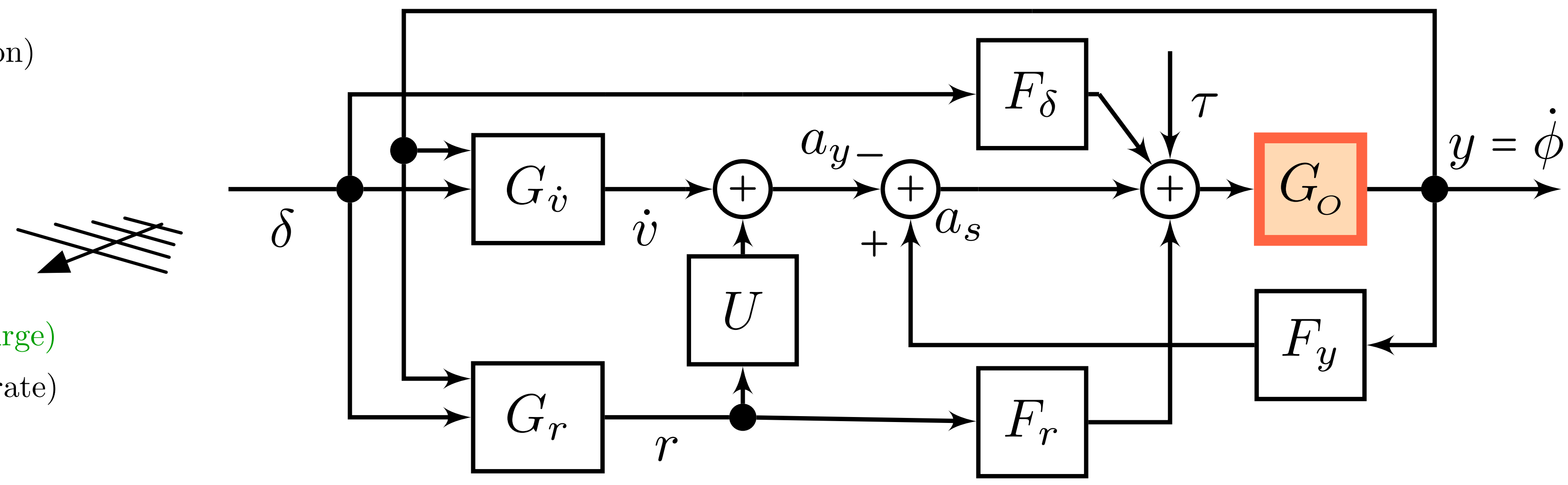


Example



Example





- The roll subsystem can be described by

$$(A_x + Mz_g^2)\ddot{\phi} = -(k + Mgz_g)\phi - d\dot{\phi} + (K_{\dot{v}} + Mz_g)\dot{v} + (K_{ur} + Mz_g)Ur + K_\delta \delta + \tau$$

- Only measure the motion with an inertial measurement unit (IMU) (and the rudder angle)

$$\begin{aligned} y_{1,t} &= \dot{\phi}_t + b_{1,t} + e_{1,t} \text{ – roll rate} \\ y_{2,t} &= a_{s,t} + b_{2,t} + e_{2,t} \text{ – tangential acceleration} \\ y_{3,t} &= -r_t + b_{3,t} + e_{3,t} \text{ – yaw rate} \\ y_{4,t} &= \delta_t + e_{4,t} \text{ – rudder angle} \end{aligned}$$

- The signal $\dot{v}$ is unknown but indirectly measured by the IMU

$$a_s = z_s\ddot{\phi} + g\phi - \dot{v} - Ur$$

- The indirect model is given by

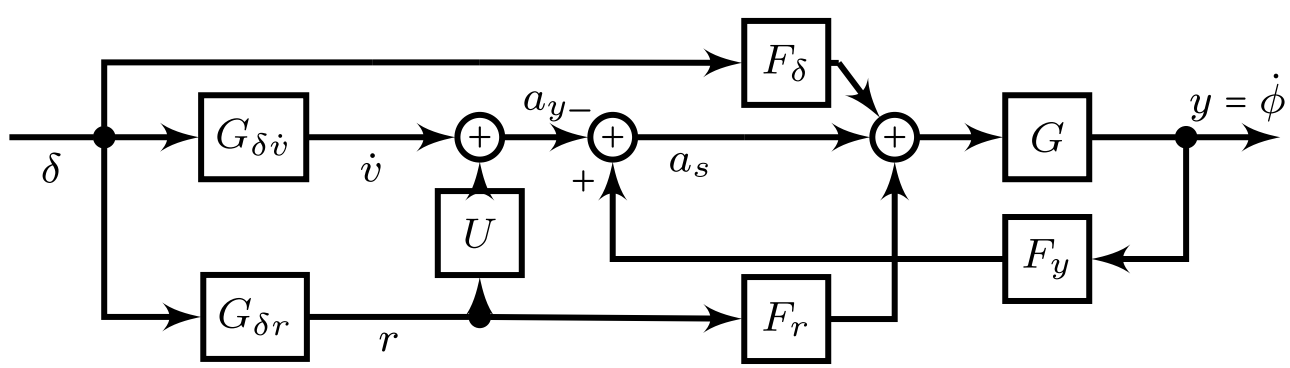
$$(A_x + Mz_g(z_g - z_s))\ddot{\phi} = -(k - K_{\dot{v}}g)\phi - d\dot{\phi} - (K_{\dot{v}} + Mz_g)a_s + K_r r + K_\delta \delta + \tau$$

Example - Estimation

- An instrumental variable method is a correlation based approach, in principle

$$\frac{1}{N}\sum_{t=1}^{N}\zeta_t\underbrace{(y_t - \varphi_t^T\vartheta)}_{\varepsilon_t} = 0$$

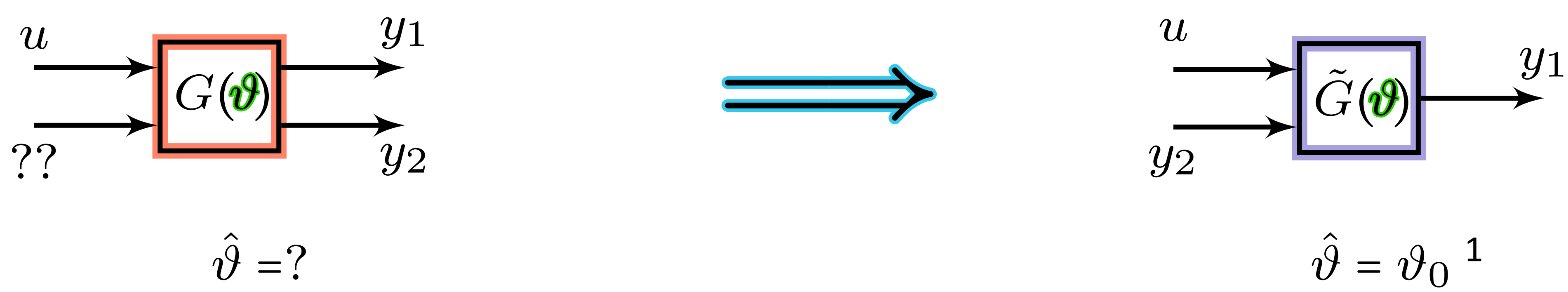
- The instruments ζ_t should be
  - correlated with the motion induced by the rudder
  - uncorrelated with the motion induced by the disturbance
- The optimal instruments would be the noise-free signals (intractable)
- The instruments are created by simulating the signals using the rudder signal



J. Linder et. al. Modeling for IMU-based Online Estimation of Ship's Mass and Center of Mass
In *Proceedings of the 10th IFAC Conference on Manoeuvring and Control of Marine Craft*, Copenhagen, 2015.

J. Linder et. al. Online Estimation of Ship's Mass and Center of Mass Using Inertial Measurements
In *Proceedings of the 10th IFAC Conference on Manoeuvring and Control of Marine Craft*, Copenhagen, 2015.

Conclusions



- Unknown input exist in many engineering applications
- A framework using indirect input measurements
- Several already existing models are recovered as special cases
- The benefits of using the framework

[1] Sometimes :)

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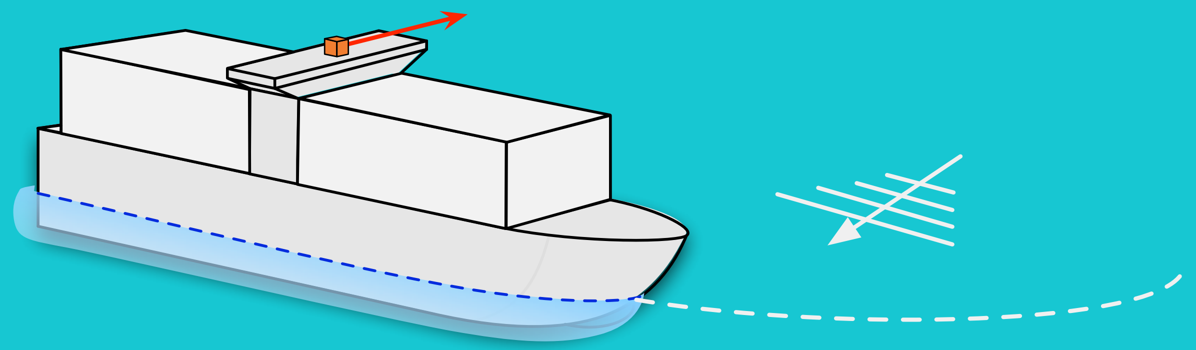

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